A MODEL FOR PREDICTING THE PROPERTIES OF THE CONSTITUENTS OF A GLASS FIBRE REBAR REINFORCED CONCRETE BEAM AT ELEVATED TEMPERATURES SIMULATING A FIRE TEST.

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ABSTRACT

This paper develops a series of expressions to predict the apparent strength and stiffness of composite rebars and the concrete matrix at elevated temperatures typical of those experienced in a standard fire test. Two methods of predicting the thermal distribution through the concrete beam at any time in a fire are compared: a semi empirical method based on experimental data and a finite element method using commercial software. The semi empirical method is found to provide the most accurate prediction and this is then used to generate expressions for the reduction in compression strength of the concrete. Expressions from earlier work, which predict the reduction in strength and stiffness of rebar with temperate are then used in combination with the calculated temperature profiles to generate equations predicting the reduction in strength and stiffness with time of the rebar encased in the concrete.

INTRODUCTION
The purpose of this paper is to develop a general method of predicting the properties of the constituent elements of a composite rebar reinforced concrete beam during a fire test, as part of a global model to predict the lifetime of the beam. The constituents in question are the FRP rebar and the concrete itself. In a previous paper the strength and stiffness properties of FRP rebar were examined under different temperature regimes, with the rebar previously subjected to a variety of representative environmental treatments.

In order to predict the properties of the rebar when they form part of a reinforced concrete beam it is necessary to be able to predict the temperature profile through the beam cross section. If this temperature distribution is known then a reasonable estimation of the FRP rebar properties at a given position and time can be obtained by referencing the properties previously reported. Knowledge of the temperature distribution will also facilitate a method of estimating the compression strength of the concrete which is known to reduce with an increase in temperature. In this work the approach of Desai\textsuperscript{1} has been taken as an approximate guide to predicting the average strength of the concrete.

Once a route to predicting the properties of the constituents of the beam has been established, as a function of time in a fire, it will then be possible to use a modified version of the fibre reinforced polymer reinforced concrete (FRP - RC) design guideline\textsuperscript{2} to predict the properties (flexural capacity, shear strength) of the complete beam as a function of time. This data will in turn allow a prediction of the time to failure of that beam under a specified service load. The model for predicting the beam lifetime is to be presented in a subsequent paper and this will be followed by papers
reporting on direct fire tests and a critical assessment of the lifetime prediction model and the real failure performance of the beam.

The ability to predict the lifetime of a beam under fire conditions is of critical importance if buildings or structures are to be made that use this type of beam.

The first consideration for this work was to assess the different methods available to predict the temperature distribution within the beam during a fire.

There are two principal options. The beam could be modelled using a finite element approach and commercial packages are available to perform this role. Alternatively a semi-empirical approach could be adopted where data obtained from real tests using embedded thermocouples can be used to construct a temperature profile.

Both approaches have been adopted. Obviously, a real fire test with a thermocouple embedded at the rebar-concrete interfaced will give the most accurate reading of the temperature at that position. However unless a large number of thermocouples are embedded this approach cannot be used to predict the temperature profile throughout the beam without some additional interpretation or interpolation of the data.

Although all real fires are different, most structural fire testing is performed under standard conditions where a test furnace is heated from room temperature to temperatures representative of a fire according to a standard heating curve (ISO - 834)\(^3\). This enables data obtained from fire testing in the literature to be used and compared in order to determine whether some simple relationships can be established regarding
the rise of temperature at specific depths in different beams subjected to a common external temperature profile. Three relevant experiments were reported in the literature on beams where the temperature at the rebar interface was measured as a function of time in a fire test. All three beams had different depths of concrete cover protecting the rebar. This literature data has been scrutinised and has allowed a general semi-empirical equation to be proposed linking temperature and time as a function of the depth in the beam. This equation is then used to predict the temperature rise at the rebar-concrete interface in the composite beam that has been designed in this work. The prediction of the semi-empirical equation and a comparable prediction produced by FE modelling are then compared to the actual result generated by a fire test on the Queen Mary beam. The semi-empirical approach is shown to provide a better prediction and is then used to predict a temperature profile across the whole cross section of the beam. This temperature profile is used in conjunction with the relationships detailed in the Eurocode\textsuperscript{4} standards to predict the effective strength of the concrete in compression. Finally, representative plots are presented which, for a given cover depth, illustrate the reduction in strength of the rebar with time, and the effective reduction in strength of the concrete.

DEVELOPMENT OF A SEMI-EMPIRICAL EQUATION TO PREDICT TEMPERATURE.

Three experiments have been reported in the literature which involved fire tests on full-scale beams where the rebar interface temperature was measured continuously
during the test by embedded thermocouples. Sakashita et al\textsuperscript{5}, and Lin et al\textsuperscript{6} conducted these experiments.

The three sets of beams were all rectangular in cross section, with FRP rebar in the tension face of the beam. In the fire tests the lower face (tension) was exposed to fire as were the two sides of the beams, but the top surface was encased in the test support apparatus and insulated from the external temperatures. The cover depths of the three beams were 30 mm, 38 mm and 105 mm.

In each case the external temperature $T$ was assumed to follow the standard temperature curve and equation required by ISO-834 (and other test standards) which is shown in Figure 1. It should be noted that all tests standards allow some slight deviation from this heating curve within tight limits.

Structures must carry their loads for a certain time when they are exposed to the ISO-834 standard fire. How long depends on the use of the structure and national building codes. The following classes are used to describe the fire resistance of structures\textsuperscript{3}:

$$\text{R30 } \text{R60 } \text{R90 } \text{R120 } \text{R180 } \text{R240}$$

Where R stands for resistance and the number for how many minutes the structure should at least carry the load.

The rebar temperature ($\theta$) /time ($t$) curves reported by Sakashita et al\textsuperscript{5}, and Lin et al\textsuperscript{6} are shown in Figure 2.
Figure 3 shows the relationship between $T - \theta$, the difference between the rebar temperature (figure 2) and the fire test furnace temperature (figure 1), $T$, versus time, $t$, from the fire tests.

It is proposed that this difference in temperature between the rebar interface temperature and that of the external fire follows a common relationship for all three experiments after 30 minutes of exposure. No attempt is made to include the first 30 minutes of the temperature history in this model. The form of the relationship is given by

$$T - \theta = Ae^{-\beta t}$$  \hspace{1cm} (1)

Where $T =$ furnace temperature

$\theta =$ interface temperature,

$A =$ empirical constant,

$t =$ time in minutes,

$\beta =$ is obtained from the gradient of $\ln(T - \theta)$ versus ($t$) derived from experimental data

The term $\beta$ is a curve fitting parameter that can be derived from the experimental data and incorporates the relationship between temperature and the depth of the cover.

Plotting $t$ versus $\ln(T - \theta)$ allows $\beta$ to be defined from the gradient of the best fit line as is depicted in Figure 4 for each concrete cover. From the best fit line equations for each concrete cover depicted in Figure 4, the values of the intercept $A$ are very
similar in each case and an average value for \( \ln A \) is 6.643 giving \( A = 767 \). This gives

\[
T - \theta = 767 \exp^{-\beta t} \tag{1a}
\]

The slopes for each line in figure 4 provide a value for \( \beta \) of 0.0011, 0.0017 and 0.0033 for cover depths of 105, 38 and 30 mm respectively. Plotting \( \beta \) against cover depth, \( c \), figure 6, reveals a relationship of the form:

\[
\beta = a \exp \left( \frac{b}{c + d} \right) \tag{2}
\]

Curve fitting software suggests that the values for \( a \), \( b \) and \( d \) of

\[
\begin{align*}
    a &= 0.001 \\
    b &= 7.602 \\
    d &= -23.623
\end{align*}
\]

The furnace temperature, \( T \), at any time can be obtained from ISO-834 equation:

\[
T = 345 \log(8t + 1) + 20
\]

We can combine this relationship with equations 1a and 2 to produce an overall equation for rebar temperature in a concrete beam during a fire test, with respect to time \( t \) and concrete cover \( c \) as:

\[
\theta = (345 \log(8t + 1) + 20) - 767 \exp\left(-\frac{7.602}{0.001 \exp^{-23.623}}\right) t 
\]

\( \tag{3} \)
The semi empirical equation, fitted to the experimental data using the appropriate value of $\beta$ is shown in Figure 6. The proposed equation appears to be a good approximation to the experimental data for all three data sets over the period of interest (after 30 minutes).

As part of this programme of work fire tests were performed on full-scale beams with dimensions 350mm x 400mm x 4250mm. The full details of these tests and the design of the beam itself will be presented in a future paper. However at this stage all that is necessary to report is that the concrete cover over rebar was 70 mm from the tension face of the beam.

**FE MODEL FOR THE EVALUATION OF THE TEMPERATURE PROFILE**

The alternative method for predicting the temperature distribution in the beam is to use Finite Element analysis. A commercial package FEMLAB was used with relevant data on the concrete properties obtained from in-house data obtained from nominally identical samples$^8$.

For the purposes of this analysis the composite rebars were ignored and the beam was considered to be a wholly concrete structure.

**Mathematical formulation**

The problem of heat transfer into a concrete beam during fire has been analysed by many workers and this section summarise their work$^6,7$. In a real fire, or a fire test,
heat will flow to the surface of the structure exposed to fire by means of radiation and convection. Then heat will be transferred internally away from the surface by means of conduction. Because of the time dependency of the gas temperature, this heat transfer is classified as a “transient” temperature analysis problem.

The Fourier transient heat conduction second order partial differential equation for flow in an homogenous medium is represented for the two-dimensional situation as follows

\[
k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + Q = \rho C \frac{\partial T}{\partial t}
\]

(4)

where

\[ k = \text{thermal conductivity of the material (in W/m deg } ^\circ \text{C)} \]
\[ T = \text{furnace temperature (in deg } ^\circ \text{C )} \]
\[ Q = \text{internally generated heat (in W/m}^3 \text{))} \]
\[ C = \text{the specific heat of the material (in J/kg deg } ^\circ \text{C)} \]
\[ \rho = \text{the density of the material (in kg/m}^3 \text{))} \]
\[ t = \text{the time variable (minutes)} \]

In problems of structures exposed to fire, the internally generated heat component \( Q \) is not active. The transient heat flux at the boundary conditions can be constant heat flux, or variable convection and radiation heat fluxes. This is represented by the following equation:

\[
q_c + q_r = k \left( \frac{\partial T}{\partial x} l_x + \frac{\partial T}{\partial y} l_y \right)
\]

(5)
The convection heat flux can be expressed as follows:

\[ q_c = \psi (T - T_s) \gamma \]  \hspace{1cm} (6)

where

\( \psi \) = convection factor, (in W/m deg C)

\( \gamma \) = convection power

\( T_s \) = average surface temperature, deg C

The radiation heat flux can be expressed by Stefan’s equation:

\[ q_r = S (\alpha_s e_f T^4 - e_s T_s^4) \]  \hspace{1cm} (7)

where

\( \alpha_s \) = absorptivity of the exposed surface

\( S \) = Stefan-Boltzmann constant, \((5.67 \times 10^{-8})\) W/m deg K

\( e_f \) = emissivity of the flame

\( e_s \) = emissivity of the surface

In most heat transfer calculation methods, Equation (6) is simplified by the use of a resultant emissivity as follows:
\[ q_s = e.S(T_s^4 - T_f^4) \]  \hspace{2cm} (8)

where \( e \) is the resultant emissivity of the surface and fire which ideally should be temperature dependent.

The boundary condition for the solution of Equation 4 is.

\[-k \frac{\partial T}{\partial n} = h_c (T_s - T_f) + eS(T_s^4 - T_f^4)\]  \hspace{2cm} (9)

in which \( n \) is the direction of heat flow normal to the boundary,

\[ h_c = \text{the convection coefficient} \]

**Analysis of temperature development in beams**

In this work a model is proposed based on FEMLAB software in order to simulate the temperature contour inside the beam at each exposure time. It is assumed that the material is homogenous and there is no cracking during the fire. Assuming that heat flow across the exposed boundary of the section is caused by both convection and radiative mechanisms.

The model outline is given as follow.

In domain equation of the FEMLAB (equation 10) the below values were substituted.

The cross dimension of the beam assumed in the analysis was 350mm x 400mm.

\[ \rho \cdot c \cdot \frac{\partial T}{\partial t} - \text{div}(k \cdot \text{grad}(T)) = 0 \]  \hspace{2cm} (10)
where
\[ \rho = \text{concrete density measured as 2331 kg/m}^3 \]
\[ c = \text{heat capacity} = 1140 \text{ J/kg °C} \]
\[ k = \text{thermal conductivity measured as 1.53 W/m °C} \]

In Equation 10 the expressions \( grad \) and \( div \) are defined as follow. The expression \( 'grad' \) is gradient of a scalar function which will be used for scalar function \( \phi(x,y,z) \) which is continuously differentiable with respect to its variable \( x,y,z \) throughout the region, then gradient of \( \phi \) written \( grad \phi \), is defined as the vector

\[
grad\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k
\]

Note that, while \( \phi \) is a scalar function, \( grad \phi \) is a vector function.

The expression \( div \) is divergence of a vector function.

The boundary condition for top surface is taken
\[ n.(k.grad(T)) = 0 \text{ zero heat flux in normal direction where n is normal vector.} \]

Boundary conditions for other surfaces are as follow
\[ n.(k.grad(T)) = e.S.((Tambient)^4 - (Tsurface)^4) \]

\[ S = \text{Stefan-Boltzman constant, (5.67 x 10}^8 \text{ W/m deg K)} \]
\[ e = \text{resultant emissivity of the flame} = 0.94 \]

\( Tambient = \text{furnace temperature} \)
\( Tsurface = \text{beam surface temperature} \)
T = dependent variable at surface

*Tambient* was interpolated, throughout the simulation in the following,

Initial condition was when \( T_0 = 20^\circ C \) at time = 0

\( h \) = convection coefficient = 0

The result of the simulation is presented in Table 1. In the model, the temperature is measured at 41 equidistance points. Results are also presented in Figure 7.

**Comparison between the two methods**

The results for the temperature-time relationship for the rebar in a beam with a 70 mm cover of concrete, predicted by the FE model and the semi-empirical equation are presented in Figure 7. This also includes the actual temperature curves recorded in our tests on a reinforced beam with a cover depth of 70 mm. It can be seen that the semi-empirical equation provides an excellent prediction for the experimental data. The finite element analysis is less accurate and underestimates the actual temperature encountered in the test.

In some respects this is a surprising result given that the semi-empirical analysis must contain some inaccuracies. The beams used to derive the equation are real three dimensional structures with different dimensions. The analysis does not take into account explicitly the fact that heat flows into the beam from both the sides and base, although this is covered implicitly by the fact that the data is from real specimens. The Finite element analysis does directly take into account the size and shape of the beam. However the FE analysis also has to rely on many assumptions, particularly related to the transient nature of heat flow at the surface of the beams. It would no doubt be possible to modify and adjust the FE analysis to fit the experimental data, but this
would be a relatively fruitless exercise for this programme of work, as the FE analysis would then be specific to this beam and the semi-empirical analysis provides results that are sufficiently accurate for our purposes.

STRENGTH AND STIFFNESS REDUCTION IN THE CONCRETE AND REBAR

1 Concrete

The semi-empirical model predicts the temperature along a linear path into the material. It is recognised that a simple linear equation cannot predict accurately a two-dimensional temperature profile across the beam section, but given that the equation is based on real experimental data in which heat enters the beam from three surfaces of the beam it is expected that only minor errors will be incorporated in the analysis. Using the semi-empirical equation with suitable values for $\beta$ and depth in the beam, a temperature profile can be constructed across the beam from each surface. A 2D plot of the temperature profile after a given time can be constructed, as shown in Figure 9 for a beam after 90 minutes. This beam has the dimensions of the reinforced concrete beam that is to be used subsequently for fire testing (350mm x 400mm). The temperature at the corners of the beam would rise more quickly than the bulk due to the combined heat input from two surfaces but this effect is likely to be small and localised and as such is ignored. Accordingly the temperature profiles consist of orthogonal lines parallel with each exposed surface.
Desai\(^1\) has suggested that an approximate route to calculating the strength of a concrete section at elevated temperatures is to produce a weighted average of the local strength of the concrete over the section.

His approach is summarised by the following equation. The section is effectively considered as a series of equal slices with the average strength in each slice calculated by averaging the strength at the boundaries of the slice.

\[
\sigma_{cT} = \frac{1}{\text{beam}} \left[ \frac{(\sigma_{cT10} + \sigma_{cT9})}{2} \text{area1} + \frac{(\sigma_{cT9} + \sigma_{cT8})}{2} \text{area2} + \ldots + \frac{(\sigma_{cT1} + \sigma_{cT0})}{2} \text{area10} \right]
\]

(11)

The local strength properties themselves can be calculated knowing the temperature at each position and using the relationship given in Eurocode 2\(^4\) which requires the concrete compressive strength \(\sigma'\) at normal temperature and a specified concrete reduction factor \(k_c\) using the following equation (Eurocode 1992)\(^4\).

\[
\frac{\sigma'_c}{\sigma_c} = k_c = \begin{cases} 
1 & \text{for } T \leq 100 \text{ (Tin °C)} \\
(1.067 - 0.00067T) & \text{for } 100 \leq T \leq 400 \\
(1.44 - 0.0016T) & \text{for } 400 \leq T \leq 900 \\
0 & \text{or } 900 \leq T 
\end{cases}
\]

(12)

(13)

(14)

This method of calculating strength obviously has a number of drawbacks. Strength in general cannot be averaged out over a section as a structure will always fail first at its weakest point. Ideally it would be better to calculate the strength at each position, undertake an iterative process to assess the effect on local concrete failures on the
overall load bearing capability of the section and continue this process until total collapse of the section was predicted. However, in compression loading failure locally will not inevitably mean that failure will spread to other sections, and the load bearing capabilities of a failed section, surrounded by material that is stronger and intact is not clear. A more in-depth analysis is likely to introduce errors due to assumptions regarding the micromechanics of the system and is unlikely to result in a significantly improved estimate of the failure load than the Desai model. The strength calculated by this method should however be referred to as an apparent strength, rather than an absolute value of strength.

The global objective of this work is to predict the performance of a beam in a fire. It is assumed that the beam will experience the thermal insult resulting from the fire on its bottom surfaces which are loaded in tension. The relevant compression strength properties of the beam are consequently averaged across the temperature profile at the top of the section shown in Figure 9. This results, using the Desai method, equation 11, in a reduction in the apparent compression strength of the concrete with temperature as indicated in Figure 10. It should be noted that this plot of strength reduction depends on the temperature profile and hence only applies to this specific rectangular beam.

The relative reduction in concrete beam strength ($k_c$) in terms of the exposure time in fire can be represented using equation 15.

$$k_c = \frac{\sigma_c'}{\sigma_{c,0\min}}$$  \hspace{1cm} (15)
where $\sigma'_{c0\text{min}}$ and $\sigma'_{ct}$ are the apparent concrete strength at 0 minutes and t minutes respectively and $k_c$ is the exposure time reduction factor for concrete beam strength. The proposed reduction factor $k_c$ as a function of time from Figure 10 can be estimated using equation 16.

$$k_c = 1 - 0.0031t \quad \text{(t in minutes)} \quad (16)$$

2 Rebar

Reduction factors for the strength and stiffness of composite rebar as a function of temperature, T, were developed in an earlier paper. Equation 3 developed in this work has been shown to provide a good estimate of the local temperature of the rebar after time t, in a fire test. Combining this information has allowed us to calculate the strength of a rebar after time, t, in a fire test, i.e. at the temperature represented by time, t, at the position of the rebar in the concrete beam. The relevant data and equations are presented in table 2. This in turn allows us to plot a strength or stiffness reduction factor for the rebar encased in the concrete, as a function of time in the fire test. This is shown in Figure 11 for strength for different cover depths of concrete, and in Figure 12 for stiffness. It should be noted that the strength/stiffness reductions for the first 30 minutes are only assumptions as the temperature profiles in this period have not been modelled.

The data taken from our earlier paper relates to a particular rebar, identified as G2 rod, which is a unidirectional reinforcing bar with a vinyl ester resin matrix.

As the cover increased the rate of degradation in properties decreased. The Rebar in beams with 105 mm and 70 mm cover retained some 20% of their stiffness for 180
minutes while the rebar in a 30 mm cover beam is predicted to have zero stiffness after 150 minutes.

The equations depicted in Figure 11 and Figure 12 give the relationship between the reduction factors and the exposure time. Equations (20 to 22) for stiffness reduction/time will be used in a subsequent paper to calculate the flexural and shear capacity of the beam during a typical fire test.

\[
k_\sigma = 1 - 0.007t \quad \text{for 105mm cover} \quad (17)
\]

\[
k_\sigma = 1 - 0.0073t \quad \text{for 70mm cover} \quad (18)
\]

\[
k_\sigma = 1 - 0.01t \quad \text{for 30mm cover} \quad (19)
\]

\[
k_E = 1 - 0.0044t \quad \text{for 105mm cover} \quad (20)
\]

\[
k_E = 1 - 0.0046t \quad \text{for 70mm cover} \quad (21)
\]

\[
k_E = 1 - 0.0063t \quad \text{for 30mm cover} \quad (22)
\]

CONCLUSIONS

A semi-empirical method for calculating the temperature profile within a reinforced concrete beam was shown to be accurate for predicting properties after the initial 30 minutes of a fire test.
The semi-empirical temperature profile allows a calculation of effective concrete compression strength to be made, using a method based on averaging local strength values, as proposed by Desai. This method predicts an effective linear decrease in the sectional concrete compression strength of a beam with time in a fire test.

A combination of temperature profile data and strength/stiffness reduction factors can be used to develop equations that predict the effective properties of rebar in a concrete beam during a fire.

The relationships developed in this paper will form the basis for the development of a general model to predict failure of a concrete beam in a fire test. The results of this work and the subsequent validation of the models using full scale fire tests will be presented in future papers.

REFERENCES


\[ T = 345\log(8t + 1) + 20 \]

Fig 1 Furnace temperature curve (ISO-834)
Fig 2 Heating time – reinforcement temperature curves obtained for various concrete cover depths, c. Note that solid lines are the recorded experimental temperatures and the dashed lines are calculated using Equation (3)
Fig 3 Relationship between the difference between furnace temperature and rebar temperature (T-θ) versus time, t, from the fire test results by Sakashita et al\textsuperscript{5}, and Lin et al\textsuperscript{6}
\[
\ln(\Delta T)_{30} = -0.0017t + 6.6239 \\
R^2 = 0.9873
\]

\[
\ln(\Delta T)_{38} = -0.0033t + 6.6553 \\
R^2 = 0.9914
\]

\[
\ln(\Delta T)_{105} = -0.0011t + 6.6502 \\
R^2 = 0.9356
\]

**Fig 4** Evaluation of \( \beta \) from the gradient of \( \ln(T - \theta) \) versus (\( t \)) derived from experimental data
**Fig 5** Comparison between the T-θ versus t curve calculated from equation 3 and the experimental fire test results for each concrete cover.
Fig 6 Relationship between the curve fitting parameter $\beta$ and concrete cover depth $c$, for rectangular beams.

\[ \beta = a \exp \frac{b}{c + d} \]
Table 1 Simulation temperature data obtained from FEMLAB model

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<th>Temperature T(ºC)</th>
<th>30min exposure</th>
<th>60min exposure</th>
<th>90min exposure</th>
<th>120min exposure</th>
<th>135min exposure</th>
<th>Distance from bottom of the beam (mm)</th>
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Fig 7  Temperature profiles for different exposure times in beams with a 70mm concrete cover, generated using the FE model.
Fig 8 Comparison between two methods of temperature predictions and the real test results at the position of the rebars with cover of 70mm in rectangular beams when exposed to fire.
Temperatures, °C

Fig 9 Estimated temperature distribution in the beam at t = 90 minutes, using the semi-empirical model
Fig 10 Reduction factor for the concrete strength in the beam at the compression face versus time of exposure in the fire test.
Fig 11 The reduction in strength with time during a fire test on a concrete beam for rebar with differing concrete covers.

\[
k_{\sigma(105\text{mm cover})} = 1 - 0.007t \\
R^2 = 0.984 \\
k_{\sigma(70\text{mm cover})} = 1 - 0.0073t \\
R^2 = 0.9851 \\
k_{\sigma(30\text{mm cover})} = 1 - 0.01t \\
R^2 = 0.9873
\]
Fig 12 The reduction in stiffness with time during a fire test on a concrete beam for rebar with differing concrete covers
Table 2 Calculation of the rebar reduction factors in strength and stiffness

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Note that $\Delta T = \theta - 20^\circ C$